

Comparing the State Estimates of a Kalman Filter to a Perfect IMM Against a Maneuvering Target

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Abstract – Tracking maneuvering targets is an important problem. A study was previously performed to compare the state estimation accuracy of a Kalman filter to an interacting multiple model (IMM) for a maneuvering target. The authors defined a maneuvering index to quantify the degree of maneuvering. Their study then compared the state estimates of the two filters as a function of this index. Their results showed that an IMM provides significant improvement over a Kalman filter. That study was revisited and this paper discusses the differing results observed. Our results show that the IMM does improve overall state estimations but much less than in the previous study. This improvement is due to the smaller state estimation errors that the IMM provides over the Kalman filter during the non-maneuvering intervals, rather than the complete domination in performance of the IMM that the previous study revealed. As a result, the "0.5 rule" that the previous authors identified, should be revised.

Keywords: Maneuvering target, perfect IMM, Monte Carlo analysis, Kalman vs IMM

1 Introduction

Tracking maneuvering targets is an important problem that occurs in many domains. In 2001, Kirubarajan and Bar-Shalom (K&B) demonstrated the relative state estimation performance of a Kalman filter to an interacting multiple model (IMM) as the target becomes increasingly more maneuverable.[1][2] They introduced a maneuver index metric which quantifies the degree of maneuverability of a target. This index is discussed later. They compared the state estimation errors of these two filters and plotted the results as a function of the maneuvering index. K&B showed that the state estimation performance of an IMM was considerably better (i.e., has lower RMS error) than a Kalman filter as the target

became more maneuverable. This analysis led to their finding of the "0.5 rule" which states that the IMM performs better than Kalman once the maneuver index exceeded 0.5.[1][2] K&B also plotted the estimation errors during the intervals that the target was maneuvering and during the intervals that the target was not maneuvering. Their results showed that the IMM produces smaller estimation errors than the Kalman filter regardless of whether the target is maneuvering or not. This result seemed unexpected. While one would expect their IMM to provide smaller estimation errors during the non-maneuvering intervals, it was not clear why their IMM would perform better than the Kalman filter during the maneuvering intervals.

That K&B study was re-visited in this study and the results are discussed in this paper. There was one difference, however, in this study. Instead of comparing the Kalman filter to an IMM, we will introduce a *Perfect IMM* filter and use this filter as the comparison to the Kalman filter. Using a Perfect IMM instead of an IMM defines the lower bound on estimation errors possible from an IMM. This paper will discuss the effort to re-create the results from the original study. We will show, unfortunately, that K&B's results could not be duplicated. We will provide justification of why we believe the results we obtained are more reasonable. Our results will suggest that the 0.5 rule that K&B identified should be revised.

2 Introducing the Perfect IMM

An IMM filter is effectively a Kalman filter with multiple motion (i.e., prediction) models. At each update, the IMM runs multiple filters, one for each motion model and then combines the resulting state estimations into an overall state estimation as a weighting of the results from each filter. The weights are generally arbitrary but based on some knowledge about the behavior of the target. Although K&B define the weights used for the IMM in

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their study, we will use a *Perfect IMM (PIMM)* filter instead. A PIMM filter is a theoretical IMM-like filter that cannot be realized for any real system, but is useful for a simulated system. In an IMM, the goal is to apply the right motion model at the right time (i.e., update). In simulation, the true motion of the target is known (to the simulation), so the PIMM is told which motion model to use for each update. Thus, the selection of the correct motion model is always performed perfectly and hence its name, Perfect IMM. Since a Perfect IMM always selects the right motion model, it defines the lower bound on the estimation errors of any corresponding IMM making it useful for comparison studies of IMM performance. Furthermore, since the Kalman filter effectively defines the upper bound on the estimation errors of any IMM, we know that any real IMM will perform somewhere between these two bounds.

For this study, a two-model PIMM is used that is similar to the two-model IMM that K&B used in their study. The two models are identical except for process noise. K&B defined a low process noise for when the target is deemed not maneuvering and a high process noise for when the target is deemed maneuvering. The same process noise models were used in the PIMM. The parameters for the process noise will be given later.

3 The Target Motion Model

In the K&B study, a simplistic two-dimensional target model is used where the acceleration error distributions in the x and y directions were identical. They assumed on each update, the target randomly selects an acceleration from a distribution with a large or small variance, depending if the target is maneuvering or maintaining (nearly) constant motion during this time update. K&B also assumed that the target maneuvers were synchronized with the measurement updates. Thus, the target always starts a maneuver at the measurement update time and continues with that maneuver until the next update time. Note that aligning the maneuver to the update time is unrealistic since these events are independent of each other. Although this is unrealistic, for consistency, we use the same approach in this study. It is suspected, however, that making the maneuver times independent of the measurement update times would not have any large affect on the results of the study.

Based on these assumptions, a simple motion model can be derived as follows. Let T be the time between updates. Let a_x and a_y be the randomly selected x and y accelerations respectively, for the i^{th} update. Then, the target state for the i^{th} update would be:

$$\dot{x}_i = \dot{x}_{i-1} + a_x T \quad (1)$$

$$\dot{y}_i = \dot{y}_{i-1} + a_y T \quad (2)$$

$$x_i = x_{i-1} + \dot{x}_{i-1} T + \frac{1}{2} a_x T^2 \quad (3)$$

$$y_i = y_{i-1} + \dot{y}_{i-1} T + \frac{1}{2} a_y T^2 \quad (4)$$

K&B state that the accelerations are selected from a zero-mean distribution with standard deviation, σ_a . The type of distribution was not stated but assumed to be Gaussian.

4 Comparison Test Method

Test scenarios will be constructed identically to the ones used in the K&B study. The initial position of the target will be at (25,000 m, 10,000 m) with an initial velocity of (-120 m/s, 0 m/s). Each scenario lasts seven minutes of simulated time. During the first, third, fifth, and seventh minute, the target moves with nearly constant velocity with process noise standard deviation $\sigma_a = 0.2 \text{ m/s}^2$. During the second, fourth, and sixth minute the target maneuvers with a (fixed) process noise level that is varied from $\sigma_a = 0.4 \text{ m/s}^2$ to 10.0 m/s^2 , in increments of 0.4 m/s^2 . The comparison graphs however, plot the state estimation root-mean-squared (RMS) errors as a function of the maneuver index, λ , just as was done in the K&B study. The maneuver index is a unit-less quantity formed as the ratio of the uncertainty in the target motion over the uncertainty in the measurement. The index is defined as:

$$\lambda = \frac{\sigma_a T^2}{\sigma_w} \quad (5)$$

where σ_a is the process noise standard deviation (in each dimension), T is the time between updates, and σ_w is the measurement error standard deviation (in each dimension).

Like the K&B study, the measurement errors in the x and y dimension are assumed to both be normally distributed with $\sigma_w = 100 \text{ m}$ in each dimension. The time between updates is $T = 5 \text{ sec}$.

The results of the comparison will be summarized as a set of plots describing the position and velocity estimation RMS errors, using Monte Carlo analysis. The original study used 100 Monte Carlo iterations for each process noise standard deviation. To make the plots smoother, this study uses 1000 Monte Carlo iterations. The average RMS error, E_{avg} is:

$$E_{avg} = \frac{1}{m} \sum_{j=1}^m \sqrt{\frac{1}{n} \sum_{i=1}^n e_{ij}^2} \quad (6)$$

where m = number of Monte Carlo iterations = 1000, n = number of measurement updates per scenario, and $e_{ij}^2 = (\hat{x}_{ij} - x_{ij})^2 + (\hat{y}_{ij} - y_{ij})^2$ is the computed error of the i^{th} measurement update on the j^{th} Monte Carlo iteration.

4.1 Preparing the Kalman Filter

The Kalman filter for this study follows identically with the one in the K&B study. It is assumed that the true state of the target at time t_k is

$$\mathbf{x}(t_k) = [x(t_k) \quad \dot{x}(t_k) \quad y(t_k) \quad \dot{y}(t_k)]^T \quad (7)$$

where $x(t_k), y(t_k)$ is the position and $\dot{x}(t_k), \dot{y}(t_k)$ is the velocity in the x, y coordinates, respectively. Like the

K&B study, the covariance matrix \mathbf{R} of measurement noise is:

$$\mathbf{R} = \begin{bmatrix} \sigma_w^2 & 0 \\ 0 & \sigma_w^2 \end{bmatrix} \quad (8)$$

The plant noise matrix \mathbf{Q} is also identical to the one used in the K&B study.

$$\mathbf{Q} = \begin{bmatrix} \frac{1}{4}\sigma_a^2 T^4 & \frac{1}{2}\sigma_a^2 T^3 & 0 & 0 \\ \frac{1}{2}\sigma_a^2 T^3 & \sigma_a^2 T^2 & 0 & 0 \\ 0 & 0 & \frac{1}{4}\sigma_a^2 T^4 & \frac{1}{2}\sigma_a^2 T^3 \\ 0 & 0 & \frac{1}{2}\sigma_a^2 T^3 & \sigma_a^2 T^2 \end{bmatrix} \quad (9)$$

where σ_a is set to 0.8 of the process noise standard deviation used during the maneuvering intervals.

4.2 Preparing the PIMM Filter

In the K&B study, the IMM simply employs two different plant noise matrices; one for when the target is deemed to be maneuvering and the other when the target is deemed to not be maneuvering. The plant noise matrix for the non-maneuvering target is found using the low process noise $\sigma_a = 0.2 \text{ m/s}^2$. The plant noise matrix for the maneuvering target is found using the process noise selected during the maneuvering interval. In this study, the PIMM filter follows similarly using the two different plant noise matrices, except it is told which plant noise matrix to use at each update.

5 Results

Figures 1 through 4 are the results obtained from this study. The figures directly correspond to the ones in the K&B study.[1] Figs 1a and 1b plot the peak position estimation RMS errors and peak velocity estimation RMS errors respectively, over the entire seven-minute scenario as a function of the maneuver index. The blue dotted line plots the RMS errors from the Kalman filter. The red solid line plots the errors from the PIMM. Like the K&B study, we show that the peak estimation errors from the Kalman filter are larger than those from the PIMM filter. However, our results show the differences between these two filters are much less than what K&B obtained.

Fig. 1a shows that the peak position errors for the Kalman at $\lambda = 2.5$ is about 155 m. K&B show this peak error to be around 300 m, about twice as large. Furthermore, the figure shows the PIMM error is about 145 m, which is only ~6% improvement. The K&B study shows the IMM error to be about 180 m, about a 40% improvement. The peak errors for the velocity estimation are also different from K&B's result. Fig. 1b shows the peak velocity errors to be approximately the same between the two filters with a difference growing to only ~5 m/s. K&B show the Kalman estimation error to be nearly three times larger than the IMM error. Additionally, K&B show that the velocity estimation errors for the IMM grow to only ~12 m/s, which is about four times smaller than the errors we obtained.

Figs 2a and 2b plot the overall position estimation errors and overall velocity estimation errors respectively, over the entire seven-minute scenario as a function of the maneuver index. Fig. 2a shows the position estimation errors for the Kalman filter leveling off to ~130 m. The K&B results show the position estimation error climbing to 210 m and seemingly continuing to climb. Their Kalman filter position errors are much larger than the errors in the measurements. Their results seem suspect since this would mean that using the measurements themselves would produce better position estimates than using the Kalman filter estimates. There is close agreement (115 m vs 120 m) for the position estimates for the IMM, except our results show a clear leveling off of error while the K&B results show an upward trend. As a result, their IMM position errors seem suspect as well.

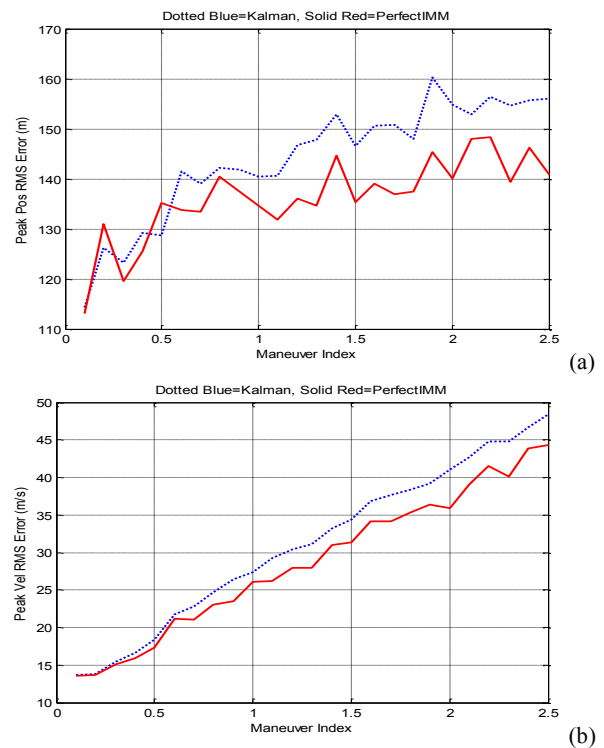


Figure 1. Base case. Peak RMS errors of Kalman versus PIMM. (a) Position errors; (b) Velocity errors.

Like the peak velocity estimation error plot, Fig 2b shows minimal difference between the two filters; again rising to a difference of only ~5 m/s. The K&B study, on the other hand, shows the velocity estimation error from the Kalman rising to twice the error of the IMM. However, our velocity errors from the Kalman filter are about twice as large as those from the K&B study, and our velocity errors from the IMM are about three times larger.

Figs 3a and 3b plot the position estimation errors and velocity estimation errors respectively, during the non-maneuvering (a.k.a., uniform motion) intervals of the scenario, as a function of the maneuver index. It is during these intervals where the PIMM is expected to outperform

Kalman since the PIMM can use the smaller process noise. As expected, the plots show the dramatic differences in the errors between the Kalman and the PIMM. The results in Fig 3a are similar to those in the K&B except they show their two filters each rise to about 15 m less. The velocity estimation errors shown in Fig. 3b are different than K&B's result, but there is agreement that the IMM estimation errors are dramatically improved over the Kalman. It is suspected that the y-axis from the K&B study is mislabeled as it shows the Kalman filter velocity errors grow to only ~4 m/s.

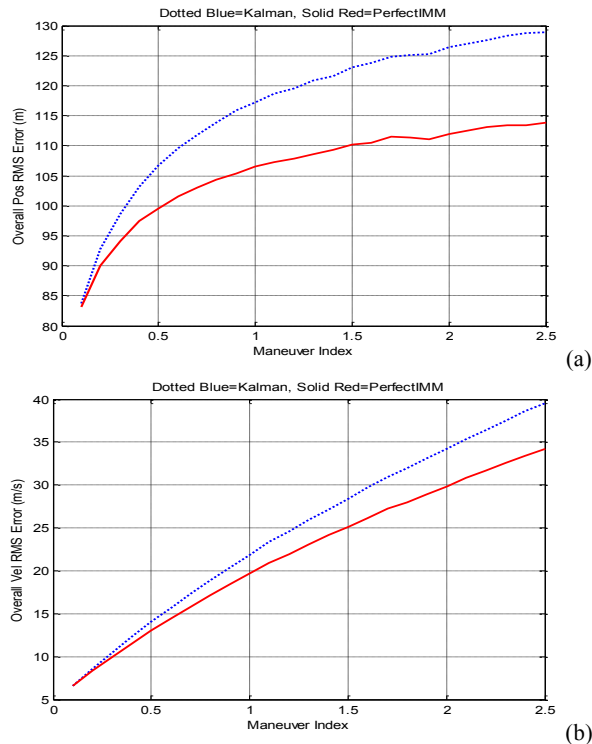


Figure 2. Base case. Overall RMS errors of Kalman versus PIMM. (a) Position errors; (b) Velocity errors.

Figs 4a and 4b plot the position estimation errors and velocity estimation errors respectively, during the maneuvering intervals of the scenario as a function of the maneuver index. Note that there is little difference between the two filters. This is what should be expected since the two filters are virtually identical during these maneuvering time intervals. The K&B study, on the other hand, shows a dramatic improvement of the IMM over the Kalman as the maneuvering index increases. In fact, they show the position errors for both filters grow larger than the errors in the measurements themselves. Their position error for the IMM grew to ~180 m and the position error for the Kalman filter grew to ~260 m. Here again their results suggest it would be (much) better to ignore the filter and just use the measurements. The fact that they showed the IMM offered dramatic improvement over the Kalman filter is questionable. Since the filters are nearly

identical during these intervals, it is unclear how they could obtain large differences between the two filters.

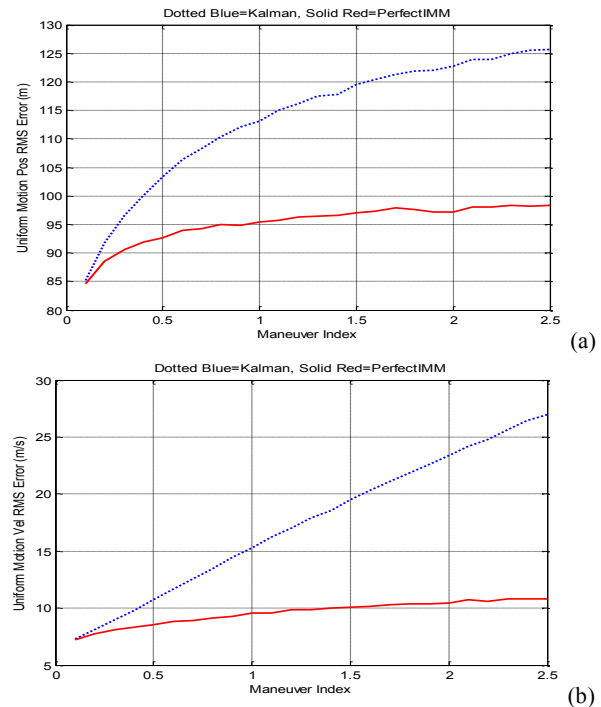


Figure 3. Base case. RMS errors of Kalman versus PIMM during the non-maneuvering intervals. (a) Position errors; (b) Velocity errors.

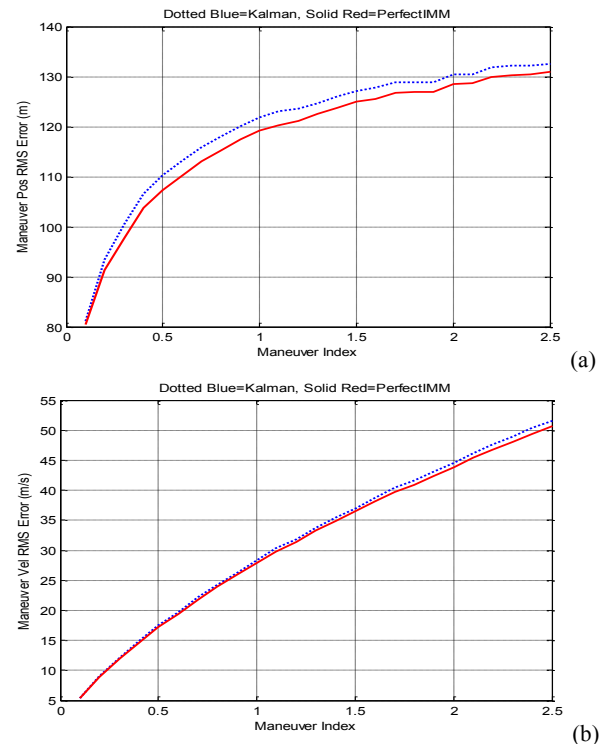


Figure 4. Base case. RMS errors of Kalman versus PIMM during the maneuvering intervals. (a) Position errors; (b) Velocity errors.

Note that since the two filters are resulting in nearly identical estimation errors, it means that the Kalman filter is producing the lower bound on the errors during these maneuvering intervals.

6 Conclusion

Based on our results, we agree with the K&B study that the IMM is superior to the Kalman filter, but only because an IMM can lower its process noise during the non-maneuvering intervals to provide tighter estimates during those times. The overall performance improvement that the IMM achieves is based on exploiting the intervals of time when the target is not maneuvering. Thus, if the target is constantly maneuvering, or at least, has only short periods when it is not maneuvering, then the IMM will offer little improvement.

It is important to recall that this study introduced and used a Perfect IMM as the comparison. As mentioned earlier, this filter provides a lower bound on the estimation errors of any IMM. The study showed that during the maneuvering intervals, the Kalman filter state estimates were as good as the PIMM, so therefore a Kalman filter provides this lower bound. It was only during the non-maneuvering intervals where the PIMM had significantly smaller estimation errors than the Kalman filter. As a result, we are not inclined to agree with K&B's '*0.5 rule*' of when it is better to use an IMM over a simpler Kalman filter. Instead we offer that the IMM will outperform the Kalman filter only if a maneuvering target does not maneuver for times long enough for the IMM to recognize that fact.

The results from this study should help guide tracking system design, especially when considering continually maneuvering targets.

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